PLASTIC DEFORMATION

BY

L. U. DE SITTER.

PART I.

Plastic deformation of the Earth's crust.

§ 1. Plastic deformation of solid matter under high confining pressures has been insufficiently studied. JEFFREYS [1]) devotes a few paragraphs to deformation of solid matter as a preface to his chapter on the isostasy problem. He distinguishes two properties of solid matter with regard to its behaviour to external forces: the Rigidity and the Strength. The rigidity being the resistance to elastic stresses, the strength the resistance to shearing forces. The strength has been surpassed when a differential force results in a permanent deformation.

Therefore the strength equals the differential pressure necessary to effect shearing or plastic deformation.

A series of interesting experiments concerning these problems by GRIGGS [2]) throws much light on the relation of strength and confining pressure. By applying high differential pressures on a block of limestone under increasing confining pressures until permanent deformation set in he showed that the confining pressure had no, or at any rate a very small, influence on the elastic properties of the material. The beginning of the deformative process is always the same elastic deformation. Even the elasticity limit, the point at which the material yields to the applied force, was hardly heightened during the increasing differential pressure. The confining pressure had to be raised above the original strength to prevent rupture (Fig. 1).

Thus the influence of the confining pressure on solid matter does not interfere with its elastic properties, nor strength, it only changes the character of its permanent deformation after the strength limit has been passed.

Hence the difference between plastic deformation and rupture must be found in different reaction to shearing forces and not in difference in strength or elastic characteristics. Also as the elastic habit of the solid remains unimpaired by raised differential pressure, some characteristics of elastic deformation will be maintained during plastic defor-

[1]) JEFFREYS „The Earth'', 2nd ed., Cambridge 1929.

[2]) GRIGGS, Deformation of rocks under high confining pressure. Journal of Geology, Vol. 44, 1936, p. 541—578.

mation. The latter thesis has been confirmed by GRIGG's experiments, where he found elastic afterworking, return, after plastic deformation „even larger than could be calculated from the elastic modulus.''

§ 2. By applying a differential pressure a set of forces is introduced; the reaction on this set prevents rupture, to the maximum strength of the material. The strength on each plane throughout the material is unvariable as long as we regard isotropic mediae only. Hence the material will yield along a plane, along which the set of

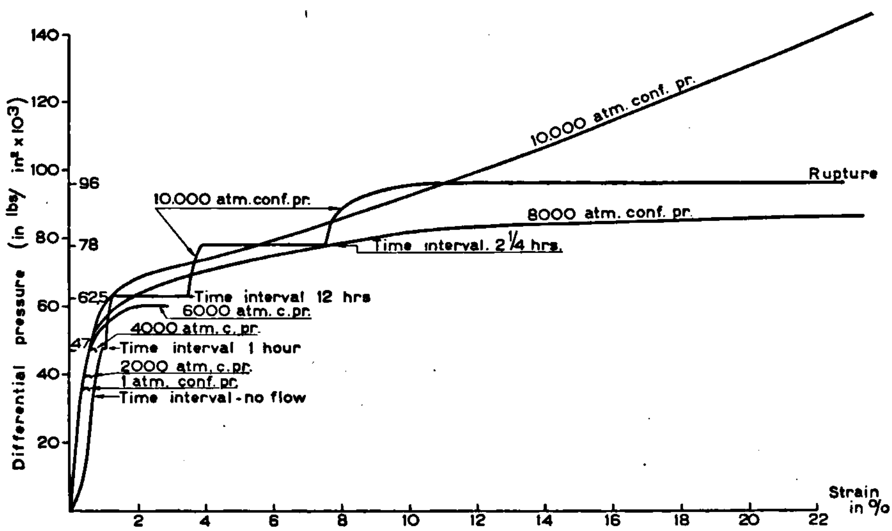

Fig. 1.

Stress-strain diagram of Solenhofen limestone under varying confining pressures. (redrawn from fig. 4 and 9, GRIGGS, „Deformation of rocks under high pressures'' Journ. Geol. Vol. XLIV).

differential force is greatest. And as the intensity of this force, e, can be calculated by

$$e = \frac{1}{2} P \sin . 2a \qquad \begin{array}{l} P = \text{differential pressure} \\ a = \text{angle between shearingplane} \\ \quad\ \text{and direction of force} \end{array}$$

e will be at a maximum along a plane with $a = 45°$ [1]).
This maximum value of P we will call the stress-limit.

Before actual movement along a shearing plane will occur another reaction of the material must be overcome. This reaction is the friction along the shearing plane. Obviously this friction depends on the character of the material, but also on the pressure exerted on the shearing plane. The friction due to the character of the material is the same for all planes in an isotropic medium, but the pressure on the plane

[1]) W. HOPKINS, Camb. Phil. Trans. 8, 1849.

is dependent on external influences, partly on the confining pressure, partly on the component of the differential force perpendicular to the shearing plane. Hence when we define the strength as the differential force needed to cause rupture (as GRIGGS does), strength must be the sum of stress-limit and shearing-limit. The shearing friction is of quite a different character to the stress-limit, although yielding it continues to act, and introduces the time element. If the movement is very slow, the shearing friction diminishes accordingly.

Or, when the confining pressure has raised the shearing friction above the strength, a differential force equalling the strength will result in a multitude of „latent'' shearing planes, but no movement will take place. A small addition to the differential force will result in a slow motion along several planes, the velocity increasing with increase of differential pressure. The latter phenomenon has been clearly illustrated by GRIGGS's experiments. Irrespective of the initial velocities and the later retardation, the velocities of plastic deformation in GRIGGS' experiments are: (Fig. 2 and 3)

Differential pressure constant at:	Shortening (strain) per ½ hour
47.10^3 lbs/in²	less than 0.01 %
62½ „ „ „	0.088 %
78 „ „ „	0.353 %
96 „ „ „	1.86 % [1]

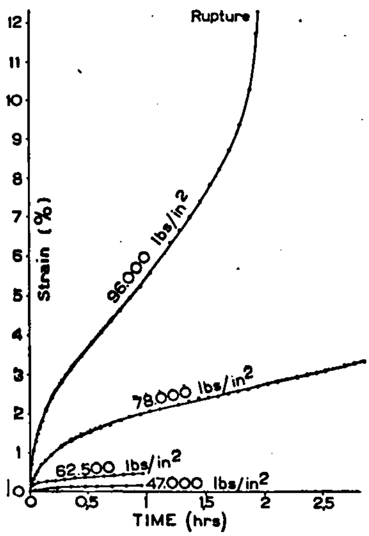

Fig. 2.

Diagram showing velocities of plastic deformations, derived from the curve with time intervals of fig. 1. (redrawn from GRIGGS, loc. cit.)

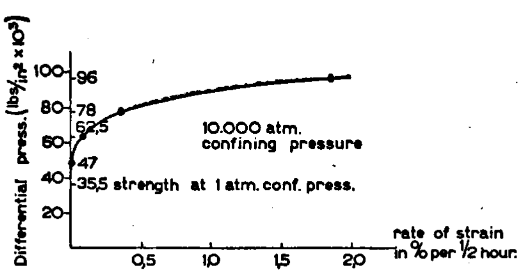

Fig. 3.

Stress-rate of strain diagram derived from the straight parts of the curves of fig. 2.

[1]) The table has been measured from the graph Fig. 2, thus the values are only approximative.

Values showing a distinct acceleration, and where the strain per time unit is not a linear function of the shearing stress (Fig. 3). The small rise of the yield point under high confining pressures, 10 % for a confining pressure 4 × the original strength, may be due to compression of the material. It would perhaps be logical to assume a greater strength for a closer packing of the constituents, but we must keep in mind that as the elastic habit of the material was not influenced by the high confining pressure, the compression can not have been very large. Consequently, I think it more probable that the rise of 10 % of the strength is only apparent, the shearing being so slow for low values of the differential stress that it can not be distinguished from elastic deformation. Theoretically the strength ,our definition of § 1, and yield value will be one and the same point on the curve (compare Fig. 3).

Another apparent contradiction between theory and experiment throws an unexpected light on the relation plastic versus elastic deformation.

GRIGGS succeeded in causing rupture of the material, even under confining pressures many times larger than the original strength, but always after considerable plastic deformation. I do not think it possible that the increasing plastic deformation itself, increased the strength. The strain curves with constant differential forces showed an initial velocity of plastic deformation, which rapidly decreased until an equilibrium was reached i.e. the straight part of the curves. However, before rupture occurred we find an acceleration of this constant velocity. The retardation of the initial velocity may be due to the continuation of the elastic deformation, or continuation of the compression, during the beginning of the plastic deformation. When the strain curve became straight, the equilibrium between the viscosity and the differential pressure had been attained. The final acceleration before rupture is probably due to the finite dimensions of the experimental block of limestone i. e. to the geometrical circumstances. This continuation of the elastic effect explains the enlarged elastic afterworking, mentioned above. Also the fact that by a large acceleration of the differential force higher ultimate strength could be reached than by a slow acceleration, points to the same continuation of elastic effect during plastic deformation.

However in geological processes the acceleration of the tangential pressure will be extremely slow, and the dimensions practically infinite. At a certain depth below the surface there will be found a confining pressure slightly larger than the strength at the surface, compensating the strength at that depth, slightly increased by compression. Tangential folding forces may also slightly increase the strength, so that we may have to go a little deeper still to find a confining pressure at which rupture becomes impossible. As soon as the tangential force has surpassed the strength limit, plastic deformation will set in, but as the material never looses its elastic habit the plastic deformation may possibly still be governed by certain laws of elasticity.

§ 3. Our inferences lead us to a distinction of matter in three classes viz.

(1) solid not plastic matter, having rigidity and strength, shearing friction small in comparison with its strength.

(2) plastic solid matter, having rigidity and strength, shearing friction large in comparison with its strength.

(3) Fluids having no rigidity, strength nor shearing friction.

Therefore, if the strength is small, the shearing-friction need not be large to allow plastic deformation e. g. as with wet clay, shoemakers wax etc.

The distinction „brittle'' and „ductile'' are properties of the material. As we have seen, a brittle material can be made to behave plastically by high confining pressure i. e., the shearing friction is enlarged by increasing the confining pressure. We can call the original shearing friction, inherent to the kind of material, the „viscosity''. An increase of confining pressure does not affect the strength, but certainly increases the viscosity.

The different reaction of fluids to deformative stresses may be due to the fact that the resistance offered by viscosity and strength is so small that no compression takes place at all. The elastic properties that direct the force in all other cases are thus left out.

The above mentioned classes can now be subdivided viz:

1 *a.* brittle solid matter fragile: low viscosity < some strength
b. brittle solid matter strong: low viscosity < great strength

2 *a.* ductile solid matter e.g. metals: high viscosity ± = great strength
b. plastic matter hard e.g. pitch: high viscosity > low strength
c. plastic matter soft e.g. butter: low viscosity > some strength

3 fluids: strength + viscosity less than compressibility.

With high confining pressures groups (1) *a* and *b* can be brought into the range of groups (2) *b* and *a*.

Thus the relation viscosity versus strength determines whether the material will rupture or be plastically deformed, but all matter belonging to the groups (1) and (2) will start their deformation by laws of elasticity, as all have an active rigidity.

JEFFREYS recently [1]) arrived at a similar conclusion. We quote from p. 21:

„These considerations suggest that when a solid is undergoing flow we must regard the distortional stress as composed of two parts, one proportional to the strain and resisted by rigidity, the other proportional to the rate of increase of strain and resisted by viscosity''.

A theoretical study of the phenomena of plastic deformation we find in the report on viscosity and plasticity by J. M. BURGERS [2]). Plasticity is regarded as relaxation of elastic stresses. An elastically deformed body contains potential energy, stored in the molecular lattice. We quote from p. 15 ... „A molecule, which thus possesses a certain energy with respect

[1]) H. JEFFREYS, Earthquakes and mountains, London 1935.

[2]) First report on viscosity and plasticity, Chapter I, by J. M. BURGERS, Verh. Kon. Ac. van Wet. Amsterdam 1935.

to its neighbours, may be thrown out of its position into one of smaller potential energy, the lost potential energy will be dissipated into heat. If such a change of position, combined with loss of potential energy, occurs repeatedly, the potential energy of the whole system will decrease. This will bring about a decrease of the internal stresses of the system, in other words the elastic stresses in the body relax." And from p. 16 ... „We may further imagine a case in which the exterior forces acting upon the body are such ,that equilibrium requires a constant value of elastic stresses. Then we shall obtain that every time the interior stresses have relaxed under the influence of thermal agitation, the equilibrium is disturbed; consequently the exterior forces will cause an increase of the deformation to such a degree that the interior stresses shall have regained their original value, so that the equilibrium will be restored. As, however, the relaxation process goes on permanently, this phenomenon will continuously repeat itself and thus we arrive at a state of continuously increasing deformation, or a state of flow under action of constant exterior forces."

These inferences explain the simultaneous action of elastic and shear stresses observed in GRIGGS' experiments. The value for a coefficient of viscosity deduced from the above mentioned theory is dependent on the thermal activity, i.e. the temperature.

The retardation of elastic deformation must be the result of an internal friction, not to be confused with the flow viscosity. This property is called the „firmo-viscosity". The property of solids to maintain this elastic habit during plastic flow is called the „elastico-viscosity". Finally the strength is regarded as a yield value of the elastico-viscosity.

§ 4. So far we have only considered the influence of increasing confining pressure. An increase of temperature, such as must be assumed in deeper layers of the earth's crust has a more widespread influence. Both viscosity and strength are lowered by rising temperature, but not necessarily at the same rate.

When the material is plastic under certain temperature, and the rise of temperature lowers the viscosity at a greater rate than the strength, the same material might rupture under the new conditions because the viscosity resistance has fallen below the strength. Thus a material belonging to group 2 *c* may be transferred to group 1 *a*. This is probably what happened in the experiments by KUENEN [1]) when in his attempts to imitate the buckling of the earth's crust, the raising of the temperature had the unexpected result that insead of the elegant curves of the harder material, often thrust planes were formed in the softened material.

A further rise of temperature must finally result in the melting of the material, the total loss of strength.

§ 5. As we have seen in paragraph 2 and 3, the shearing friction is dependent on the pressure on the shearing plane. This pressure is the sum of the confining pressure and the component of the distortional

[1]) PH. H. KUENEN, Leidsche Geol. Med. VIII, p. 169—214, 1936, fig. 12.

stress perpendicular to the shearing plane. Consequently the shearing friction is smallest for all planes parallel to the elastic stress, and greatest on planes perpendicular to that stress, both principal planes of stress.

In other words shearing would prefer planes parallel to the elastic stress direction if means to effect shortening with such shearing were possible. If the length of the beam is large compared to its thickness these means are found by the formation of a fold. The shearing takes place parallel to the upper and lower surface, i.e. the original direction of the elastic stresses. How the shearing stresses are formed is not easy to determine. Their direction and values must be found by a theoretical treatment of the problem of plastic deformations. The outcome of this theory will probably be that concentric folding approximately or truly is the geometrical result of the orientation of the distortional stresses. Thus the difference between plastic deformation of fluids and solids is that the distortion of an element in the fluid is independent of its situation relative to the whole body, whereas in solids the situation of that element is all important, as the distortional stress is different in size and direction for every point. The theory of plastic deformation must determine this size and orientation for every point at any moment. In a former paper [1]) it has been shown that the shear of concentric folding is restricted to certain parts of the fold, and the amount of shear depends on the dip of the strata at a certain point.

Such plastic buckling is only possible if the length of the beam parallel to the stress is large compared to the thickness. This condition is fulfilled in the case of the earth's crust, but not in GRIGGS' experiments. It is not necessary that the arching starts by elastic buckling, any surface irregularity will suffice to direct the distortional stresses.

VENING MEINESZ [2]) determines the buckling limit and calculates a stress of 5.25×10^4 kg/cm^2 necessary to effect elastic buckling [3]) He agrees that obviously this is more than the crust can stand, and concludes that the crust must be layered to be able to buckle. All his calculations are based however on elastic buckling. The crust *will* give way long before that calculated stress is reached, but not by rupture or by elastic buckling of smaller units, but by plastic flow, in the way described above. The maximum stress needed to effect plastic flow will not be appreciably more than the strength of the material as tested in laboratory conditions, and even may be less [4]).

[1]) L. U. DE SITTER, Leidsche Geol. Med. Vol. VIII, p. 161—168, 1936.

[2]) VENING MEINESZ, Gravity expeditions at Sea, Vol. II, 1934.

[3]) The same equation for the buckling limit has been used by JEFFREYS „The Earth'', 2nd edit., 1929, p. 288. From the eq. JEFFREYS concludes that evidently „The crust can transmit the stresses perfectly for any distance, and failure takes place where the stress difference first reaches the strength of the rocks''. The only conclusion the equation justifies.

[4]) The strength as determined in the laboratory has a rather arbitrary value. We measure the P of HOPKINS eq., whereas the e represents the proper strength of the material. In a fold the position of the planes of shear varies considerably, and the deduction of the distortional stresses from the tangential pressure in such complicated conditions can not be predicted so easily. As plastic flow does not take place at the same moment throughout the whole body, a concentration of distortional stress at one point may lead to a more favorable relation between e and P as in HOPKINS eq., where e is max. = ½ P.

From the same set of equations VENING MEINESZ deduces provisionally the wave length and even the shape of the arcs to be expected in the Earth's crust. As his equations are based however on elastic deformations it seems highly improbable that they can be applied to this problem of plastic deformations of the crust.

It may be possible that the shape and size of a fold is indeed dependent on the elastic properties of the body, because plastic flow of a non Newtonian liquid (a liquid where the ratio shearing force-rate of strain is not linear) retains its elastic habit, but until the theory of plasticity has reached a stage where we can predict the orientation and the value of the shearing forces at any point of the deformed body and at any time, such mathematical deductions as those of VENING MEINESZ are not warranted.

§ 6. The strength of rocks as determined in the laboratory varies from 7 to 40 kg/mm^2 [1]). Granite is taken as having a strength of 8 kg/mm^2 by JEFFREYS. As the confining pressure increases with depth in the Earth's crust at the rate of 1 atm. in 4 metres, the confining pressure at 3.2 km depth would be sufficient to allow plastic folding of granite. Sedimentary rocks, specially those of younger age, have much smaller strength. The plastic state will be reached for those rocks even at less than one km depth. Those shallow depths must not surprise us, as we know that sediments, which never have been buried at greater depths than ½ km, have undergone perfect plastic deformation in folds.

A sedimentary series of rocks has a natural tendency to increase in strength downwards, due to compaction and cementation, but as the confining pressure also increases downwards, the whole series can behave as a plastic solid. The strength of the whole heterogeneous series is determined by the strength of the constituents. As JEFFREYS (The Earth, footnote p. 181) explains, the strength of the heterogeneous series will be larger than its weakest constituent, but smaller than its strongest member. In assuming that the confining pressure everywhere surpasses the strength of every member of the series, a reasonable assumption for most sedimentary series [2]), the yield point of the sedimentary series is determined by its composite strength. Theoretically we may thus regard the whole series as a homogeneous body having a certain stress-limit equal to this composite strength. This explains why in simple structures, where anticlines and synclines remain seperate tectonic units, the behaviour of widely different rocks is the same. In general a cross section through a simple anticline does not show any difference in tectonics for limestones, shales, sandstones or coallayers. All conform to the same geometrical shape, whatever their specific physical properties.

Besides the strength of the material, which thus determines the yield point of the whole series, we have to consider the viscosity. During plastic deformation the strength is no longer of importance, the rate of strain is determined by the viscosity. Perhaps it is this „elastico-viscosity"

[1]) LANDOLT BÖRNSTEIN, Physikalische tabellen.

[2]) Only such formations as very young reef limestones will possibly be exceptions to this assumption.

that influences the size and shape of the fold. Unfortunately this property of solids has received very little attention. Only experiments such as those of GRIGGS and BRIDGMAN [1]), executed under high confining pressure, can help us to elucidate this problem.

The behaviour of the material in the earth's crust under tangential pressure is influenced by the increasing temperature and confining pressure on the strength and on the viscosity of the rocks. As we have seen the increase of the confining pressure has no or very little influence on the strength. The strength will remain unimpaired until a depth is reached where a beginning of melting sets in. The thickness of this layer of unimpaired strength must be at least 40 km, otherwise the crust would not be able to carry the enormous weight of the large mountain systems [2]). The viscosity is raised by the increasing confining pressure but decreased by the increase of temperature. The results of this conflict can not be measured, but it is probable that below the upper layer of 40 km thickness we shall find a zone, where the strength lies considerably below the viscosity. However, we can imagine that the increasing temperature has the power to decrease the viscosity to such an extent that at some depth the viscosity falls below the little strength the material still possesses. If that were true we should get below that second zone of flow a zone of fracture, comparable to the conditions realized in KUENEN's experiments mentioned in § 4. Such a zone of fracture at great depth can perhaps be identified as the seet of the deep focus earthquakes [3]).

Such conjectures are however of little value as long as the combined influence of pressure and temperature on viscosity and strength and rigidity is so little known. A first step to a better foundation of a theory of buckling of the earth's crust could be, failing a good theoretical basis, to interpolate these influences from laboratory experiments.

PART II.

Outline of theory of plastic deformation in folds.

§ 1. Every theory on plastic deformation of the Earth's crust, either seeing it as a unit, or treating the more limited subject of sedimentary folds, must consider the principles of plasticity. However, an exact physical theory on plastic deformation, which can be applied to the formation of folds is lacking. Hence the curious fact that authors, geological or geophysical, who realize the necessity of a mathematical-

[1]) GRIGGS, loc. cit.
BRIDGMAN, Shearing phenomena at high pressure etc. Journ. Geol., Vol. XLIV, 1936.
[2]) JEFFREYS, The Earth, sec. ed., 1929.
[3]) This suggestion can be compared to that of GUTENBERG, „Structure of the Earth's crust'', Bull. Geol. Soc. Am., Vol. 47, p. 1606. GUTENBERG assumes that the viscosity at that depth is large enough to allow an accumulation of stress, which suddenly released causes the earthquake.

physical basis for any theoretical tectonic theory, have to stop their calculations when the plastic deformations enter their discussions. As long as the deformative forces stay below the elasticity-limit (stress-limit, strength) we can sail on the compass of the theory of elasticity, but as soon as permanent deformations occur, this theory leaves us stranded on the shore of unexplored territory.

In the next paragraphs we will try to deduct the principle of concentric folding from the fundamental properties of solids set forth in the first part of this paper.

Our belief that concentric folding is the leading principle of folding tectonics is founded on the observation of folds in nature, and not on the theoretical foundations we trie to give to our theory.

It is a well known fact that anticlines become steeper the further we penetrate down in the core. Complications in the shape of faults and thrusts will be found in the core and not far out in the flanks. Obviously this phenomenon is incompatible with a mode of deformation hereafter called „simple deformation''. A simple thickening of the strata will always tend to flatten the structure downwards (compare Fig. 7). The origin of the complications of the core of an anticline must be sought in the fact that the strata adhere as long as possible to the principle of concentric folding. As soon as the curvature of the upper layers can not be followed any longer by the lower strata without loss of volume, the latter have to adopt themselves to the space left in some other way.

Thus we have two important observations on which the concentric folding principle is based. In the first place the direct observation of concentric folding in folds, e.g. in oil field anticlines, in the simple Jura folds, and in experimental work; and in the second place the fact that all anticlines tend to get steeper and more complicated downwards and inwards.

Concentric folding as such has been generally recognised as a leading principle. The way of section construction of Busk [1]) and other methods used by oil companies are founded on this principle.

In the core of the anticline, however, they all make a serious error: they disregard the necessity of retention of volume for every layer. As soon as a centre is cut out in the construction of Busk, the volume of the layer constructed below this centre is less than that of the higher layers. This error of the construction method results in a flattening of the structure downwards, a result contradictory to natural circumstances. In nature the retention of volume is attained by the development of a thrustplane or by a change in the direction of the minute shearingplanes.

I am convinced that if we knew how to apply the laws of concentric folding and retention of volume in the proper way we should be able to explain many important features of folding tectonics; e.g. the size of folds, the asymmetry of anticlines, the development of thrustplanes, their position and shape, attenuation of steep flanks and perhaps

[1]) H. G. Busk, „Earth flexures'', Cambridge 1929.

even the general distribution of folds in a geosynclinal basin. There is no doubt, however, that we ought to understand in the first place the fundamental principles of these laws before we can apply them.

Although I realize that a complete mathematical theory is necessary, being unable to supply one myself, I will endeavour to set forth some fundamentals in plain but perhaps inadequate language. I sincerely hope that my statements will induce abler men to put the whole subject on a stabler footing. The problem is fascinating enough, but seems to offer great mathematical difficulties.

§ 2. Deformation of liquids and solids.

Deformation of a solid body under deformative stress can be divided in three successive stages:

(1) Compression; particals of the material are pressed together, resulting in a decrease of volume, both of the total volume and of the volume of a unit element.

(2) Elastic deformation; total volume unchanged, but volume of unit elements is altered.

(3) Plastic deformation; no change in volume, even in that of small elements.

The phases (1) and (2) need not be subsequent phases but may overlap one another, and are closely connected.

The limit between (2) and (3) is called the elasticity-limit or stress limit. Beyond such stress permanent deformations may take place. In ordinary hard solids (stones etc.) the material will fracture when the stress limit has been passed, but when the same material is put under high confining pressure plastic deformation can be effected without rupture.

Deformation of an ordinary liquid (water) does not pass the successive phases of the solids but is limited to the plastic phase only, compressibility being very low.

However there is a great fundamental difference between the mechanism of plastic deformation of solids and liquids, which is clearly demonstrated by the shape of the deformed body (Fig. 4).

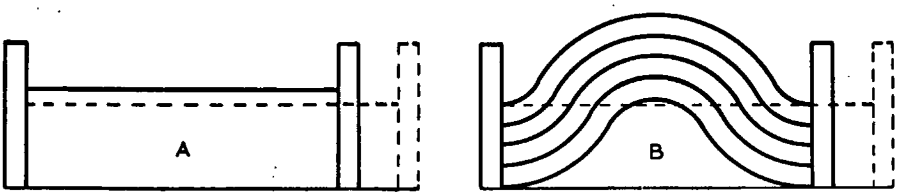

Fig. 4.

Types of deformation.
A = liquid, with raised surface, B = plastic solid, folded.

The mechanics of fig. 4a we will call simple deformation, these of fig 4b shearing (see also fig. 5) [1]).

The resistance which the material offers to the deformative stress is the internal friction or viscosity. In other words in liquids the resistance is small, and in solids the high viscosity is supported by the strength, the latter constituting a yield value (threshold limit) to the whole process.

The difference in reaction of liquids and solids in plastic condition must be explained by this difference in reaction of the medium to deformative stresses.

Simple deformation can be described as the deformation of a square (unit element) to a rectangle. (Fig. 5a).

Shear can be described as the deformation of a square to a parallellogram. (Fig. 5b).

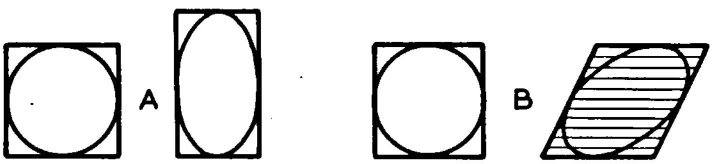

Fig. 5.

Deformation of the unit element.
A = liquid, simple deformation, B = solid, simple shear.

Obviously, both ways of deformation can be represented by a simple elongation in one direction, as illustrated by the change of shape of the circle to the ellips. However, the difference between 5a and 5b is the mechanism of the deformation. During deformation in the case of 5a the distance of particles relative to one another is always changing, whereas in 5b the distance of particles parallel to the shearing plane remains unaltered. The fundamental difference between 5a and 5b is that the particles in a liquid have no fixed position relative to one another, no definite shearing planes can or will be formed. In a solid the position of one particle to the next can not be arbitrarily changed, to a definite stress direction belong definite shearing planes.

The deformation of liquids of fig. 4a is composed of those of fig. 5a, and the deformation of solids as in fig. 4b is composed of unit elements deformed as in fig. 5b (see fig. 6).

Deformation is then effected along definite shearing planes. Another function of the strength as threshold value is the preference for increasing development of one fold instead of general deformation of the whole body, as happens in fluids. Somehow it is easier to continue folding where it once started until some other agent, probably gravity,

[1]) To effect the deformation of fig. 4b the length of the beam must be large compared to its thickness; if not the shearing will result in a change of shape similar to that of fig. 4a, but different in internal mechanism. The shearing planes are then no longer strictly parallel.

puts an end to it. In other words, the property of strength determines the kind of deformation. In the case of a viscous liquid we reach an intermediate stage. The resistance of the viscosity diminishes with the rate of deformation, hence for very low stresses, the resistance is very low and pure liquid deformation will result. The increase of viscosity resistance with increase of stress, however, will result in a kind of

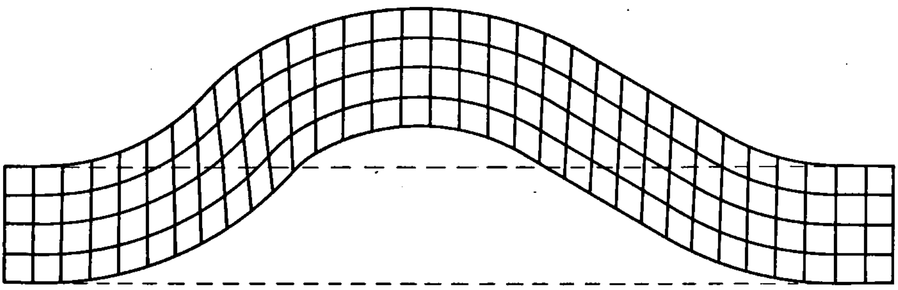

Fig. 6.

Concentric fold, unit elements deformed by simple shear parallel to bedding planes.

strength for high differential stresses. This apparent strength has the same influence as the proper strength in so far as it constitutes a threshold value in restricting as much as possible the deformation to one fold, but does not necessitate definite shearing planes. The result will be a fold formed by simple deformation, where we are not able to designate everywhere definite parallel shearing planes (fig. 6) [1].

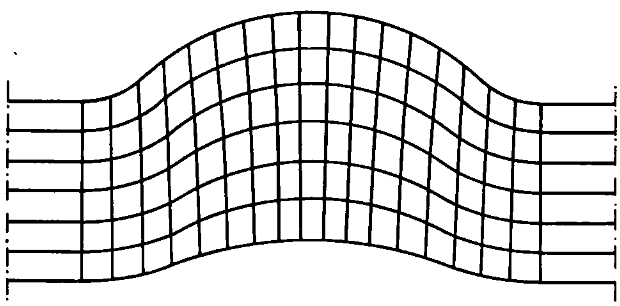

Fig. 7.

Composite fold, Unit elements deformed by shear and simple deformation.

The volume of the unit elements is constant, which together with the decrease of dip downwards determines the pattern. In the lower layers we find, predominating simple deformation, in the upper layers simple shear. The combination of fluid and solid properties, cf. the apparent

[1]) The pattern of the unit elements was sugegsted by one of a series of experiments by Ph. Kuenen, which have not been published yet.

strength of a viscous fluid under excessive deformative stress, is illustrated by the deformation of the unit elements. The quicker we try to deform a viscous material as pitch the more it reacts as a solid.

With a blow of a hammer we can break the fluid material. This is the same phenomenon which according to GUTENBERG [1]) accounts for deep focus Earthquakes.

Material having a definite strength will be permanently deformed by means of definite shearing planes. The arrangement of the planes may result in a change of total shape similar to that of fig. 4a, as for instance in GRIGGS' experiments, but the deformation of the unit element remains that of the type of fig. 4b.

Possibly a material having a small strength put under high distortional stress will act as a highly viscous liquid and show the intermediate type of deformation of fig. 7.

But in geological processes the strength of the material is very high. The distortional stress will slowly increase until the stress limit has been reached. As soon as this limit has been passed plastic deformation sets in, absorbing the original very slow rate of increase of stress. Hence, in considering deformation of folds of the Earth's crust we are not concerned with distortional stresses considerably exceeding the stress limit, and the intermediate kind of deformation may be left out of the discussion of the fundamental principles.

§ 3. In the foregoing paragraph we have established the fact that the deformation of the unit element of solids must be effected by simple shear.

The arrangements of these parallelograms, the deformed squares,

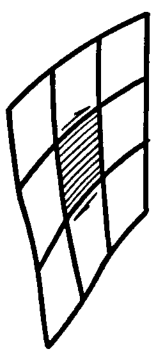

Fig. 8.

Parallelism and curvature of shearing planes.

will determine the outward shape of the whole fold (fig. 6). The confining pressure of the surrounding rock will prevent in the earth's crust the formation of voids. The compressibility of the solid material

[1]) GUTENBERG „Structure of the Earths crust'', Bull. Geol. Soc. Am., Vol. 47, 1936.

(i.e. the loss of volume) can be disregarded, because (1) the first stage of deformation process has already compressed the material, (2) the compressibility is very small from the beginning and accordingly cannot play any important part in the geometry of the final fold[1]).

These considerations lead to the general rule that in any solid the shearing planes formed by distortional stress will tend to as much parallelism as possible.

If we consider a unit element surrounded by similar elements, any deviation of this rule in a direction perpendicular to the shearing planes of the central unit (fig. 8) would either result in compression or tension. In the direction of the shearing planes of the central unit the parallelism need not be retained, there are no objections to curvature of the shearing planes.

The rule of the parallelism is a direct result of the property called strength. If this property is missing, the rule is no longer applicable. The structure of fig. 7, a combination of simple deformation and shear, compared with the pure shear of fig. 4b and 6 illustrates this rule.

§ 4. The phenomenon of shear is generally viewed from the equation of COULOMB-HOPKINS, or its modifications. The fold of Fig. 9 shows how a fold could develop along such shearing planes.

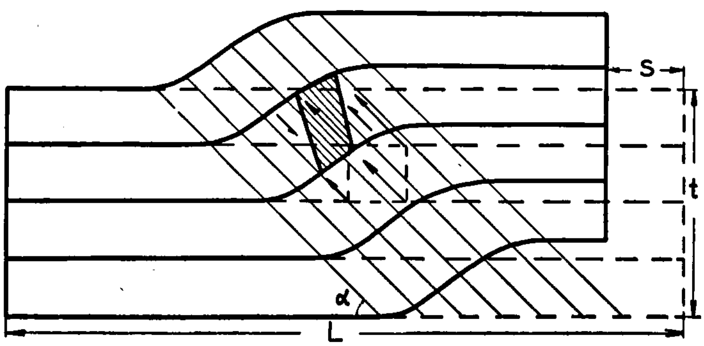

Fig. 9.

Oblique shearing (flexure).

The shearing planes are parallel, and neither the total volume nor the volume of a unit element has changed.

When D is the total shear, that it to say the sum of the shear of all unit elements, and t the thickness, s the shortening and α the angle of the shearplane, then

$$D = \frac{2\,st}{\sin . 2\,\alpha}$$

or a maximum of shortening with a minimum of shear has been reached with a α of 45°.

[1]) Compaction of shales by folding stresses may influence the shape of the fold.

However this it *not* the way sedimentary rocks are folded in general, but it is identical with a flexure, which any time may be replaced by a fault.

Concentric folding, which seems to be the leading principle in the mechanism of folding is effected in a different way. In the latter case the shear planes are parallel to the surface as in fig. 10 (and 6).

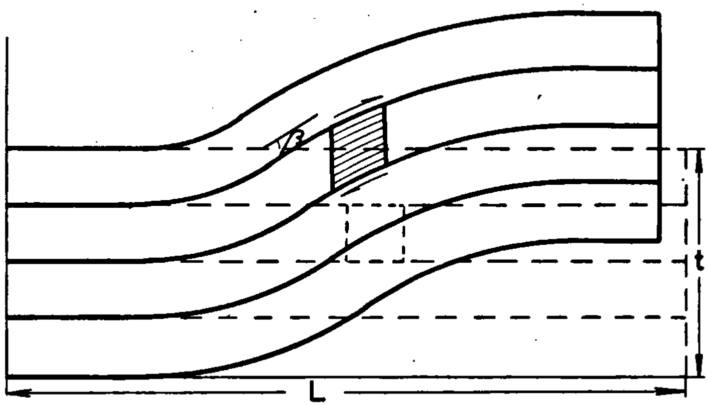

Fig. 10.

Concentric folding.

The conditions of parallelism of the shearing planes and maintenance of volume are fulfilled. The total amount of shearing, D, is again the sum of the shearing of all unit elements. If we conceive the whole curve of the fold, L, as a series of tangential circle arcs, l_1, l_2 etc. with angles α_1, α_2 etc., then for a layer of the unit thickness.

$$D_{\text{one horizon}} = \tfrac{1}{2} (l_1 \alpha_1 + l_2 \alpha_2 + \text{etc.}) \quad \text{where } l_1 + l_2 + \text{etc.} = L.$$

If pure parallelism has been maintained all horizons must be concentric circles and the lengths L are equal for every horizons, thus

$$D_{\text{total series}} = \tfrac{1}{2} t\, (l_1 \alpha_1 + l_2 \alpha_2 + \text{etc.})$$

In the simplest case, when the curve consists of two circle arcs with contrary curvatures and maximum dip $= \beta$

$$D = \tfrac{1}{2}\, t\, L\, \beta$$

§ 5. The way of shearing of fig. 9 is derived from the theory of fracturing. Instead of one shearing plane along which the rupture occurs, we have imagined a multitude of identical planes with small movements. Moreover I cannot find any reason to explain the supposed dispersion of the total movement on many planes. Also a serious objection against this kind of folding can be raised, which at the same time explains the necessity of the concentric folding principle. We know that the elastic property is not lost by passing into the plastic stage.

The orientation of the stress in the whole body must be parallel to the surface. Shearing planes crossing the stress direction would immediately loosen the elastic tension. This sudden release of tension is identical with the momentous and sudden character of the fracturing phenomennon.

In other words, it seems logical to ascribe the principle of concentric folding to the necessity of forming shearing planes parallel to the direction of elastic stress.

In the first part of this paper (I, § 3), we quoted BURGERS, explaining plastic deformation as a relaxation of elastic stresses. Hence logically relaxation can only take place parallel to these stresses. The property of retaining elastic tension is the same as the property called rigidity by JEFFREYS.

We have learned thus to recognise three properties of solids (1) the strength (2) the elastico-viscosity (3) the rigidity and the parts they play in a deformation of a solid. The preference for folding to faulting shown by rocks under sufficient confining pressure must then be ascribed to the fact that under those conditions of strength, viscosity and rigidity, less deformative stress is needed for concentric shearing than for faulting (or flexuring), because the same amount of shear (D in the equation) is followed by a larger strain percentage in case of faulting than in case of concentric shearing.

Apparently it is not the profitable effect (amount of shear) that determines the way of deformation, but the actual stresslimit, and the latter apparently is lower for concentric shearing than for faulting. The proper definitions of these properties (strength, viscosity, rigidity) are difficult to word and the theory of plastic deformation cannot be started or completed unless the mutual relations of these characteristics have been properly understood. In the absence of this, this outline will remain a superficies without contents.

Leiden, Mei 1937.